A Possible Cosmological Explanation of why Supersymmetry is hiding at the LHC

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If one is not ready to pay a large fine-tuning price within supersymmetric models given the current measurement of the Higgs boson mass, one can envisage a scenario where the supersymmetric spectrum is made of heavy scalar sparticles and much lighter fermionic superpartners. We offer a cosmological explanation of why nature might have chosen such a mass pattern: the opposite mass pattern is not observed experimentally because it is not compatible with the plausible idea that the universe went through a period of primordial inflation.

There are many good reasons to believe that the Standard Model (SM) is not the ultimate theory of nature since it is unable to answer many fundamental questions. One of them, why and how the electroweak scale and the Planck scale are so hierarchically separated has motivated the Minimal Supersymmetric extension of the Standard Model (MSSM) as the underlying theory at scales of order the TeV. However, the CERN collaborations ATLAS [1] and CMS [2] have recently reported the discovery of a boson, with mass around $(125-126)$ GeV, whose branching ratios are fully consistent with the SM Higgs boson.

Such a value of the Higgs mass implies, in the context of the MSSM, a top squark heavier than a few TeV, which in turn causes a fine-tuning at the percent level. Therefore, the lack of evidence for sparticles at LHC suggests that either low-energy supersymmetric theories are fine-tuned or (some of the) sparticles are much heavier than the weak scale. This would leave gauge coupling unification as the only, albeit indirect, evidence for low-scale supersymmetry [3] (besides the presence of dark matter provided by the lightest neutralino).

On the other hand, gauge coupling unification can be achieved for heavy scalar sparticles and much lighter fermionic superpartners (gauginos and higgsinos). For instance, choosing the fermion masses near a TeV, as dictated by reproducing the correct dark matter abundance, reproduces successful unification independently of the masses of scalar sparticles. This is the underlying idea of split supersymmetry [4] where sfermion masses can be as large as the unification scale and, more recently of mini-split supersymmetry [5], where the mass of the Higgs correlates with the mass of the sfermions which have to be in the $(10-10^5)$ TeV range [6] (unless $\tan\beta \lesssim 3$).

The question we would like to ask in this short note is: why is supersymmetry hiding in LHC searches or, put in more modest way, why are sfermions masses m so much heavier than the gaugino and higgsino masses $M \sim \mathcal{O}(1-10)$ TeV? The answer we wish to offer is based on cosmological considerations: the opposite mass pattern is not observed experimentally because it is not compatible with the universe as we know it. The logic relies on two well-accepted ideas.

First, there are many (over a hundred) flat directions in the field space of the MSSM. It may happen that some combination of the squark and/or slepton mass-squared parameters get negative at some scale below the Planck scale when running through the Renormalization Group Equations (RGE's) from the weak scale up. This may happen if the sfermion masses are lighter than the gaugino masses, leading either to the appearance of unacceptable color/charge breaking minima (provided that nonrenormalizable superpotentials are present which can lift the direction) or to unbounded from below directions in the effective potential for the squark and/or slepton fields.

The situation here is fairly analogous to what happens for the effective potential of the SM Higgs field: for a top quark mass large compared to the Higgs and gauge bosons masses, the one-loop top quark contribution to the effective potential will dominate the others and drive the coefficient of the quartic term negative for very large values of the Higgs field thus destabilizing the effective potential and making our vacuum a local, but not global, minimum [7–10].

Secondly, one of the basic ideas of modern cosmology is that there was an epoch early in the history of the universe when potential, or vacuum, energy dominated other forms of energy density such as matter or radiation. During such a vacuum-dominated de Sitter stage, the scale factor grew (nearly) exponentially in time. During this phase, dubbed inflation [11, 12], a small, smooth spatial region of size less than the Hubble radius could grow so large as to easily encompass the comoving volume of the entire presently observable universe. If the universe underwent such a period of rapid expansion, one can understand why the observed universe is so homogeneous and isotropic to high accuracy. Inflation has also become the dominant paradigm for understanding the

initial conditions for the large scale structure formation and for cosmic microwave background anisotropy.

A key feature of inflation is that, during a period of de Sitter characterized by a nearly constant Hubble rate H, all scalar fields with mass smaller than the Hubble rate H are inevitably quantum-mechanically excited with a nearly flat spectrum. The consequence of it is that excessive fluctuations of the scalar field parametrizing the flat direction during inflation would destabilize our current vacuum. If the Hubble rate is large enough during inflation and the sfermion masses are too light, then the classical value of the flat directions field is pushed above their instability point towards color/charge breaking minima which would either not correspond to the observable universe we live in (and in fact this reasoning would extend to the whole inflated region, much beyond our observed universe) or stop inflation after a few e-folds. Both possibilities are not acceptable. One can therefore reasonably conclude that the reason why sfermion masses are heavier than gaugino masses is because the opposite situation is not compatible with the idea that the universe suffered a period of inflation without destabilizing our well-behaving vacuum.

Let us elaborate now a bit further on these ideas and let us consider, out of the many MSSM flat directions, the $u_1d_2d_3$ (subindices indicate the quark family) flat direction parametrize by the field ϕ. Along this particular direction (as for many others) the coefficient of the quartic term ϕ^4 is vanishing for all scales μ, and the renormalization group improved potential reads

$$V(\phi) = \frac{1}{2}m^2(\phi)\phi^2, \tag{1}$$

where we have conveniently set the renormalization scale $\mu = \phi$ in order to make the one-loop logarithms small. The RGE for m^2 is given by

$$\begin{aligned}\mu\frac{\mathrm{d}m^2}{\mathrm{d}\mu} &= \frac{1}{8\pi^2}\left[-16g_3^2M_3^2 - \frac{8}{3}\,g_1^2M_1^2\right.\\ &+ \left. 2h_b^2\left(m^2_{\tilde{q}_L} + m^2_{\tilde{b}_R} + m^2_{H_1} + A_b^2\right)\right],\end{aligned} \tag{2}$$

where g_1 is the standard $U(1)$ coupling, M_i are gaugino masses, h_b is the bottom quark Yukawa coupling and A_b is the bottom quark trilinear mixing parameter, and all parameters are running. If $\tan\beta$ (the ratio of the two neutral MSSM Higgs vacuum expectation values) is not too large, h_b is small and the term proportional to h_b^2 in the above equation can be neglected. Furthermore, if one takes $M_1 \leq M_3$, as usually results in GUT models, the effects of the term proportional to g_1^2 in Eq. (3) is also negligible. This leads to the solution

$$\begin{aligned}m^2(\mu) &= m^2 - \frac{2}{\pi^2}g_3^2(M_3)M_3^2\ln(\mu/M_3)\\ &\times \left\{\frac{1+3g_3^2(M_3)\ln(\mu/M_3)/(16\pi^2)}{\left[1+3g_3^2(M_3)\ln(\mu/M_3)/(8\pi^2)\right]^2}\right\},\end{aligned} \tag{3}$$

where all masses on the right-hand side are physical (propagator pole) masses evaluated at the low-energy scale M_3 and the only undertermined factors is $g_3^2(M_3)$, which depends on the full spectrum of the superpartner masses and the initial condition $\alpha_3(M_Z) = 0.12$. To obtain $g_3(M_3)$ we have assumed a common supersymmetric threshold at M_3, but the results are almost unsensitive to this simplifying assumption. Eq. (3) delivers an instability scale

$$\Lambda \simeq M_3 \exp\left[\frac{\pi^2 m^2}{2g_3^2M_3^2}\right], \tag{4}$$

technically defined to be the scale at which the potential has a maximum. Solving Eq. (2) numerically without approximations, it turns out that for $m \lesssim 0.7M_3$ [13, 14] the quadratic mass of the flat direction can run to negative values, signalling the appearance of an instability at some energy value Λ smaller than the GUT scale. We find that $\Lambda \simeq 10^{10}\,(10^6)$ GeV for $m \simeq 600$ GeV and $M_3 \simeq 1$ (2) TeV. It is also clear from Eq. (3) that in the opposite case, $m \gtrsim M_3$, no instability occurs.

The color conserving minimum, although metastable, has a lifetime longer than the present age of the universe and can survive both quantum tunneling and the effects of high temperatures in the early universe, causing the color/charge breaking effects to be in practice not dangerous [14]. This holds in the post-inflationary stage though.

What may happen during inflation if sfermion masses m are lighter than gaugino masses M? As we mentioned above, a period of inflation destabilizes the (color/charge preserving) vacuum and may even stop inflation. The process of generating a classical flat direction field ϕ configuration in the inflationary universe can be interpreted as the result of the Brownian motion of the field ϕ under the action of its quantum fluctuations which are converted into the classical field when their wavelengths overcome the Hubble length.

The best way to describe the structure of the fluctuations is provided by the stochastic approach in which one defines the comoving distribution of probability $P_c(\phi, t)$ to find the field value at a given time at a given point [15]. The subscript c serves to indicate that P_c corresponds to the fraction of original comoving volume filled by the flat direction field at the time t. The comoving probability satisfies the Fokker-Planck equation

$$\frac{\partial P_c}{\partial t} = \frac{\partial}{\partial\phi}\left[\frac{H^3}{8\pi^2}\frac{\partial P_c}{\partial\phi} + \frac{V'(\phi)}{3H}P_c\right]. \tag{5}$$

To solve this equation we assume that the Hubble rate is approximately constant during inflation. Furthermore, we assume that the flat direction field is initially localized at $\phi = 0$, $P_c(\phi, 0) = \delta_D(\phi)$, and study its evolution. Eq. (5) can be solved by separation of variables

$$P_c(\phi, t) = \sum_{n=0}^{\infty} c_n e^{-\left(\alpha V + \frac{H^3 a_n t}{8\pi^2}\right)} g_n(\phi), \qquad \alpha = \frac{8\pi^2}{3H^4}\ , \quad (6)$$

where g_n and a_n are the eigenfunctions and the eigenvalues of the equation $g_n'' - \alpha V' g_n' = -a_n g_n$. We are interested in the case in which the Hubble rate is much larger than Λ. In this case the potential term can be neglected since $\alpha V' g_n'/g_n'' \lesssim (m^2\Lambda^2/H^4) \ll 1$, once we use the condition $\phi < \Lambda$. One can easily get convinced that, for $H \gg \Lambda$, the variance of the field is given by $\langle\phi^2\rangle \sim (H^3 t/4\pi^2)$. This means that after a short time $t \sim \Lambda^2/H^3 \ll H^{-1}$, the typical value of the field is of the order Λ and the instability region is easily accessible (this happens much before the evolution of the flat direction might be blocked by other non-flat direction [16]). This is also reflected in the computation of the survival probability P_Λ for the flat direction field to remain in the region $\phi < \Lambda$: it is exponentially suppressed at times $t \gg H^{-1}$ (for technicalities see Ref. [7])

$$P_\Lambda(t) \equiv \int_0^\Lambda \mathrm{d}\phi\ P_c(\phi, t) \simeq \frac{2}{\pi} e^{-\frac{H^3 t}{32\Lambda^2}},\ \ (H \gg \Lambda). \quad (7)$$

This is valid for $t \gg \Lambda^2/H^3$ and therefore it happens very rapidly, justifying the assumption of neglecting the time dependence of the Hubble constant.

Since most models of inflation predict a large number of e-folds $N \simeq Ht$, we conclude that a de Sitter stage with Hubble rate H larger than Λ will lead to a current universe which looks completely different from ours if sfermion masses are lighter than gaugino masses: a universe where either color and charge are broken or a universe where inflation took place only for a few e-folds before the negative energy density of the runaway direction took over the vacuum energy density driving inflation. It is indeed important to keep in mind that truly runaway directions may exist: because of the non-renormalization theorem for the superpotential, even operators consistent with all symmetries need not appear in the superpotential; in certain instances the absence of the gauge invariant operators which could lift the flat direction can be guaranteed by an R symmetry; directions of this type are therefore exactly flat in the supersymmetric limit. Only soft terms contribute to the potential along such runaway directions.

We acknowledge that – so far – our logic has two weak points:
1) we have assumed that the Hubble rate during inflation is larger than the would-be instability scale, $H \gg \Lambda$. It is interesting to note that the Hubble rate parametrizes the amount of tensor perturbations during inflation. Tensor modes can give rise to B-modes of polarization of the CMB radiation through Thomson scatterings of the CMB photons off free electrons at last scattering [18]. The amplitude of the B-modes depends on the amplitude of the gravity waves generated during inflation, which in turn depends on the energy scale at which inflation occured. Current CMB anisotropy data impose the upper bound on the tensor-to-scalar power ratio $T/S \lesssim 0.5$ [19] and PLANCK's expected sensitivity is about $T/S = 0.05$. This corresponds to a minimum testable value of $H \simeq 6.7 \times 10^{13}$ GeV [20]. A detection of tensor modes by PLANCK would indicate a large value of the Hubble rate during inflation and would therefore support the logic put forward in this note;
2) it explains why the hierarchy $m \lesssim M$ may not be cosmologically acceptable, but not why $m \gg M$.

In particular, having sfermions as massive as fermionic superpartners, $m \sim M$, seems legitimate from the cosmologically point of view. How can we explain the large hierarchy $m \gg M$? So far, we have assumed the validity of the form (1) for the flat direction potential. During inflation the potential of the flat direction might not be of the simple form (1) and mass squared of the flat directions of the form $\mathcal{O}(H^2)$ may be generated from the corrections to the Kähler potential

$$\delta\mathcal{K} \supset \int \mathrm{d}\theta\, \frac{\chi^\dagger\chi}{M_{\rm pl}}\, \phi^\dagger\phi \supset c_m H^2 \phi^2, \qquad (8)$$

where χ is a field which dominates the energy density of the universe, $M_{\rm pl}$ is the reduced Planck mass and c_m is a $\mathcal{O}(1)$ coefficient [17]. Suppose the parameter c_m is negative for at least one dangerous flat direction. This is not inconceivable as a positive contribution in the Kähler potential gives a negative contribution to m^2. If so, our logic is strengthened: if at least one of the many flat directions whose mass squared runs to negative values at some scale below the Planck scale has $c_m < 0$ and is a truly runaway direction, then the only safe possibility is that the mass of the sfermion at the low-energy scale is large

$$m^2 \gtrsim H^2 \gg M^2 \sim \mathrm{TeV}^2, \qquad (9)$$

otherwise inflation is promptly stopped and $M \sim$ TeV is the condition to have a successful dark matter candidate. This conclusion is further fortified by the fact that also gaugino masses can obtain $\mathcal{O}(H)$ corrections to their masses, thus leading to a faster running of m^2 towards negative values. There is no indication that the Hubble parameter may not be in the $(10 - 10^5)$ TeV range. If so, the condition (9) might be a natural explanation for the mini-split spectrum preferred by the recent data on the Higgs mass. If, on the other hand, tensor modes will

be soon discovered signaling a high energy scale for inflation, then the split supersymmetric spectrum will be favoured.

The condition (9) may be necessary even when $m \gg M$ if one considers the fact that multipole flat directions are present. Suppose that $M \ll m \ll H$ and $c_m < 0$ for a flat direction which this time is lifted by a nonrenormalizable superpotential containing a single field ψ not in the flat direction and some number of fields which make up the flat direction

$$\mathcal{W}_n = \frac{\lambda}{M^{n-3}}\psi\phi^{n-1}, \tag{10}$$

for some mass scale M. Examples of this type are represented by the flat direction udd which might be lifted by $\mathcal{W}_4 = uude/M$ and by the Que direction possibly lifted by $\mathcal{W}_9 = QuQuQuH_1ee/M^6$. When the flat direction gets a vacuum expectation value $\langle\phi\rangle \sim \left(c_m H^2 M^{2(n-3)}/\lambda^2\right)^{1/(2n-4)}$, then the flat direction involving the ψ field (*e.g.* the LLe direction for $\mathcal{W}_4$) will be destabilized by the associated tadpole, even if their corresponding mass squared $\mathcal{O}(H^2)$ is positive

$$V(\psi) = +H^2\psi^2 - \lambda H\psi\langle\phi\rangle^{n-1}/M^{n-3} + \cdots, \tag{11}$$

where the dots indicate higher order terms. The tadpole leads to a negative energy density $\sim M^4\left(H^2/M^2\right)^{(n-1)/(n-2)}/\lambda^{2/(n-1)}$. This energy can be easily larger than the de Sitter energy density $\sim H^2 M_{\rm pl}^2$, thus stopping inflation, if for instance $M = M_{\rm pl}$ and $\lambda \lesssim (H/M_{\rm pl})^{(n-1)/(n-2)}$. The condition (9) should be invoked to avoid such a disaster.

If $0 < c_m \ll 1$, the same conclusion can be reached. In such a case the de Sitter induced fluctuations of the flat direction can be as large as $\langle\phi^2\rangle \sim 3H^4/8\pi^2 m^2 \sim 10^{-1}H^2/c_m^2$. This fluctuation induces a tadpole in the potential of the other flat direction ψ and the resulting negative energy density is larger than the vacuum energy density driving inflation if $c_m \lesssim (H/M_{\rm pl})^{(n-2)/2(n-1)}$. For n large, this is not a tight constraint and once again the condition (9) can save the situation.

In most of the literature flat direction vacua during inflation are assumed to have an energy density smaller than the one driving inflation. In this short note we have argued that – for at least one flat direction – this assumption is not correct, then the hierarchy $m \gg M$ might be motivated from the cosmological point of view: once inflation is accepted, large sfermion masses ensure the stability of the vacuum and the universe with all the LSS as we know it. In the opposite case $m \lesssim M$ either inflationary quantum mechanical fluctuations or $\mathcal{O}(H^2)$ corrections to the sfermion masses may be harmful. It would be an indication that nature has been kind to us: she has provided a primordial period of inflation to generate the structures we observe and live in, she has hidden sufficiently well supersymmetry, still providing us with a livable vacuum, and she has provided light enough supersymmetric fermions to explain the dark matter puzzle.

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